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Mathematical Problem-Solving Processes in Students with Autism Spectrum Disorder

Hana Sotáková

Abstract: This study aims to describe how students with autism spectrum disorder (ASD) approach mathematical problems, how they process them cognitively, and what the specifics of the way they solve them are. We primarily focus on whether it is possible to find differences in the way these students approach the problems as compared to students with no known disabilities and whether we can find ASD-related differences in their problem-solving procedures. We draw on the qualitative empirical investigation that formed the basis of the author's dissertation. We worked with six upper-elementary school students (sixth- and ninth-graders) diagnosed with ASD and 12 neurotypical classmates. The results of the study show that the processing of mathematical problems in the students with ASD is influenced by the severity of the disorder as well as by the level of language and social skills. However, on the basis of the findings, we have to conclude that although we can find specific features in the behaviour of students with ASD referring to the core problems of the disorder, their cognitive processing of mathematical problems is very individual.

Key Words: Autism spectrum disorders, upper elementary school students, mathematical problems, word problems, reading comprehension, language abilities, cognitive processing

Introduction

Individuals with autism spectrum disorder have been the subject of interest for professionals in the fields of medicine, psychology, and education for the last 50 years. Because of their different social behaviour and communication, but also thanks to individuals with exceptional abilities in certain areas, there is an indication that neurodiversity can help professionals understand the cognitive processing not only of individuals with ASD but also of the general population. In fact, the variability of autism spectrum disorders covers both individuals with intellectual disabilities who are in need of constant support throughout their lives and individuals whose symptoms are mild and who, thanks to learned compensatory mechanisms, are able to lead independent lives without the need for specific support.

More and more often today, we encounter students with ASD in mainstream elementary schools, where they work according to the school's curriculum without major modifications or support measures. At present, the education of students with ASD is governed by the Education Act 561/2004 Coll. and Decree 27/2016 Coll. as effective from 1 January 2021 (Decree 27/2016, [online]). According to these legal norms, specific requirements for a student's education are identified and the degree of support measures necessary for a successful educational process is determined. As a result, this means that there is a growing need in the Czech education system to analyse the specifics of students with ASD in education.

Theoretical Background

Cognitive Processing and its Specifics in Individuals with ASD

Autism spectrum disorders are now a fairly well-known diagnostic entity, although there are still many phenomena associated with them that are not comprehended completely and that even today's experts are unable to fully explain. Autism comprises a spectrum of developmental disorders on a neurobiological basis with a high degree of hereditary factors arising at an early stage of CNS development (Hrdlička & Komárek, 2004; Thorová, 2006; Šporclová, 2018).

In the course of the research investigation, we based our findings on several theories of the specifics of cognitive processing in individuals with ASD, while respecting the neuropsychological concept of differences in the function of individual brain centres, although it should be noted that these are only framework specifics that do not fully explain the variability of autism spectrum disorders, and, moreover, may not be present in all individuals with ASD. A brief description of their characteristics, however, will help us explain the test performance of the students who were monitored and their procedures and their behaviour during the research investigation.

Neuropsychological Concepts and Cognitivist Theories

Neuropsychological theories are based on the fact that autism is caused by functional changes in the brain that occur during the prenatal period or in early childhood, thus focusing on the relationship between brain functioning and individual behaviour (Thorová, 2006). Although it has still not been possible to pinpoint the area responsible for the onset of autism, there are many findings on which some neuropsychological theories have been built. The ability to monitor brain activity also provides substantial information for explaining the basis of some previously researched theories of cognitive processing. We will therefore attempt to link them and show how they explain the specificities of individuals with ASD described primarily at the behavioural level.

Central Coherence Deficit Theory

One such theory is the theory of deficient hemispheric connectivity. According to this theory, the weak cooperation of individual brain centres is at the root of the problems of individuals with ASD, which results in the fact that an individual may have good or above-average performance in certain individual activities but is unable to integrate and connect them, thereby failing in complex activities (Thorová, 2006). This can be demonstrated in individuals with ASD, for example by their often above-average ability to put together puzzles or read letters (decode) but also by deficits in reading comprehension or explaining what is happening in the puzzle's picture.

Just et al. (2004) investigated the lack of connectivity on cortical activation and synchronization of the centres involved during sentence comprehension tasks. Their results showed, among other things, that the connectivity of individual activated centres was significantly lower in individuals with ASD than in the control group. This could explain the cause of the impaired central coherence (Frith, 2003). The theory in question points out that individuals with ASD often perform well on tests such as Raven's Matrices, the Block Design subest (WISC), or the Hidden Figures test. According to Frith, this is possible because unlike neurotypical individuals, individuals with ASD are unable to process information from the environment in a complex way as described by Gestalt psychologists (Frith, 2003). They tend not to move towards a whole form that is not a mere sum of parts but exhibits novel qualities (Hoskovec, Nakonečný, & Sedláková, 2002). Rather, they focus on parts, i.e. details that in some cases may not be important to the whole at all. Thus, even if in the given context we are talking about a weakened central coherence (deficit), it may act positively in the aforementioned tests (Frith, 2003; Happé, 1999). In practice, we see that individuals focus on insignificant details, which can qualitatively affect their ability to understand the spoken word or the text being read (Norbury & Bishop, 2002).

Theory of Mind and Deficits in Amygdala Function

The theory of mind is based on the fact that each individual creates an image of their internal states (mental states[1]) and the surrounding world in their mind in order to process them further. The concept was postulated by Premack and Woodruff in the 1970s, when their research showed that even primates have the ability to "read the mind" of another member of their species and thanks to this explain and predict the behaviour of the given individual. One of the most famous applications of the theory in psychology is the research of Wimmer and Perner (1983), aimed at the possibility of testing this ability in children talking about a wrong belief. The concept of the theory of mind is widely adopted in our environment but Sedláková (2004) points out a certain inaccuracy in the name itself. She believes that the meaning of the English word "mind" would be better described in Czech by the word "psyche". We call the given image a mental representation. Hončíková (2008) explains that mental representations can be distinguished into several levels (orders). She distinguishes the first order of representations, which is the mental image of the world around us, the second order of representations, which is the image of the inner world of an individual, and the third order of representations, which is a reflection of the second order mental representation/mental representation of another person. Baron-Cohen, Frith, and Leslie (1985) showed that children with ASD exhibit difficulties in developing a theory of mind (Baron-Cohen, Frith, & Leslie, 1985). This led to the creation of the theory of mind tests and their inclusion in the diagnosis of autism spectrum disorders (e.g. Happé, 1995).

Baron-Cohen was aware, however, that the theory of mind cannot explain all the deficits described in individuals with ASD. Therefore, he introduced his E-S (Empathizing-Systemizing) theory based on the two factors of empathy and systemization and their representation in the individual's cognitive processes (Baron-Cohen, 2009). He describes the empathy factor as a theory of mind-based ability that enables an individual to respond appropriately to another's thoughts and emotional states; the factor scores lower for people with ASD than for all comparison groups. On the other hand, the second factor, systemization[2] (as a tendency to analyse or construct systems by identifying the rules that govern the system), is significantly stronger in people with ASD than in the general population, according to Baron-Cohen. Baron-Cohen proposes the theory of deficit function of the amygdala as

1 Sedláková (2004) indicates that the mental state is the internal state of an individual, which has a psychological content and conveys the feelings of the individual or their reflections of the surrounding world.

2 In Czech this is called sorting or designating (see Petráčková & Kraus, 1995)

a neural region responsible for social behaviour to explain these difficulties (Baron-Cohen et al., 1999).

The Theory of Executive Function Deficits

Sally Ozonoff and her colleagues approach executive functions in a broader sense, i.e. as all functions that direct our behaviour and actions leading them to a certain goal. These include, for example, cognitive flexibility, emotional control, planning, organization, and self-evaluation (Ozonoff, Pennington, & Rogers, 1991). Difficulties in these areas are encountered in patients with frontal lobe damage, so the explanation for the nature of similar difficulties in individuals with ASD relates specifically to impaired frontal lobe function. Individuals with ASD show impairment in these functions regardless of their level of intellectual ability.

While the theory of mind mainly explains specifics in social behaviour and communication, this theory tries to capture the aspect explaining the different functioning and cognitive processing of individuals with ASD in general. Rajendran and Mitchell (2007) state that it mainly focuses on explaining specific manifestations in individuals such as difficulties in cognitive shifting, a desire for constancy, perseveration, or deficits in self-regulation. Thorová adds that another deficit caused by executive function disorder includes problems in the ability to form mental representations of objects or activities, which leads to an inability to form a course of action, a system of steps leading to a goal (completion of an activity) (Thorová, 2006).

Tatyana B. Glezerman brings interesting insights to the whole discussion. She points out that a comprehensive neuropsychological view is not only about recording brain activity but we must also consider the introspective findings of individuals with autism, the products of the activities of those individuals, and case analyses of individuals with severe symptoms of disorders who are unable to talk about their states, thoughts, and emotions. She proposes the possibility of a new approach to the study of the brain in relation to autism, whereby brain abnormalities are first detected and then changes in brain activity are monitored on the basis of the reorganization of the brain's functional systems (Glezerman, 2013). We were able to observe a similar approach in a study by Baron-Cohen (1999).

Mathematical Skills in Individuals with ASD

Mathematical ability is often considered a less problematic area for individuals with ASD, so not many experts or research studies address it. The focus tends to be on individuals with above-average mathematical abilities or savants.[3] Glezerman (2013) shows that Kanner and Asperger already laid some groundwork for the concept when they described cases of their patients with special talents who were extremely interested in mathematics and capable of complex numerical operations without being taught. *"His mother said that already early on in his schooling, he had set out a problem for himself – which is greater than 1/16 or 1/18 – and was able to solve it easily. When someone jokingly asked him, just to test his abilities, 'What's 2/3 of 120?' he immediately answered correctly, '80.' Similarly, he surprised everyone by showing that he understood the concept of negative numbers, which he had obviously taught himself and which others noticed when he remarked that 3 minus 5 equals 2 below zero. By the end of his first year, he could easily solve word problems such as – If two workers do a job in a certain amount of time, how much time will it take six workers to do it?" (cf. Asperger, 1991, p. 45).*

This boy, who was able to solve complex problems intuitively, was unable to learn basic arithmetical operations at school. Asperger also describes cases of other patients who were unable to use the usual procedures taught in school but used their own original processes, which were often based on very complex, intricate mathematical operations (Asperger, 1991). Glezerman emphasizes that these methods are not random; they are often based on a different approach to numbers and arithmetic. We can see that people with autism attribute properties or characteristics to numbers – they have emotional meaning for them. According to her, this is in turn related to the fact that the processing of numbers takes place in the right hemisphere of the brain, unlike in the majority population (Glezerman, 2013).

The idea of above-average mathematical abilities in individuals with ASD is supported by studies based on the assumption that many prominent mathematicians and physicists in history (e.g. Einstein, Gauss) can be found to have had symptoms consistent with autism spectrum disorder (James, 2010). On the other hand, Tasha M. Oswald and her colleagues (Oswald et al., 2016) charted the mathematical performance of 27 adolescents with ASD in their study and the results of their standardized tests showed mathematical aptitude in only one adolescent, while 20 had average results, and the performance of six of them was even consistent with a mathematical disorder. They thus warn against a template view of mathematical ability in individuals with ASD.

[3] Savant syndrome is a combination of exceptional mental or artistic abilities (exceptional memory, mathematical or artistic talent) combined with mental retardation (in the classic definition) or autism. (Velký lékařský slovník (The Great Medical Dictionary) online)

The mathematical abilities of individuals with ASD are as variable as those of the general population and it would be too simplistic to perceive individuals with ASD (with intellectual ability levels in the average or above-average range) as being automatically gifted in mathematics. We agree with Oswald (2016) that this can be counterproductive to the point of leading to some neglect of the area. Bae, Chiang, and Hickson's study (2015) shows that while children with ASD may perform well in numerical calculations, this does not imply good performance in word problems. The children's language level and the level of reading comprehension are also important here.

Difficulties in mathematics in people with ASD can be found across the spectrum, as is the case in the neurotypical population, with the level of intellectual ability and the level of development of language and social skills obviously coming into play. In our work, we examine students enrolled in mainstream elementary schools with levels of intellectual ability in the average or above-average range. We will therefore not analyse further the difficulties specific to students with ASD in combination with intellectual disability. We would like to mention, however, that it is not easy to find a unified or comprehensive approach in the literature. Often, a multi-sensory approach is emphasized for individuals with ASD combined with intellectual disabilities, and structured learning with strong visual support is recommended (cf. Vosmik & Bělohlávková, 2010 or Fletcher, Boon, & Cihak, 2010).

King, Lemons, and Davidson (2016) build on the idea that the variability in autism spectrum disorders underpins the different requirements for identifying difficulties in mathematics. They suggest that the core problems of autism also cause difficulties in mathematics for gifted students with ASD, particularly in the areas of critical and analytical thinking when solving more challenging problems. Donaldson and Zager add that they found difficulties with processing problems in students with high-functioning autism particularly in the areas of working memory, organization of information during problem solving, comprehension of word problems, and ultimately deficits in abstraction (Donaldson & Zager, 2010). The above areas are very general and it can be said that some of them (e.g. critical thinking) are problematic even in the neurotypical population.

Comprehension of word problems can largely be related to the overall level of a student's language, communication, and social skills. Students with ASD prefer direct instruction without social context but this does not correspond to the normal school reality. For example, as reported by Rendl and Vondrová (2013), according to mathematics teachers, problems with context are considered to be word problems. They add that the main difficulty they see for students is not understanding the text and translating it into an idea of the situation and the nature of the problem at stake. The students either focus on a particular aspect of the assignment and disregard the rest of it or they manage a partial calculation but are unable to integrate it into the whole.

Here, we come to an issue we have not yet mentioned, which is motivation. Even Attwood (2005) points out that sometimes the major problem is that individuals with ASD are not motivated to solve problems if they do not find the solution simple and straightforward or if the problem is not about their favourite topic. On the other hand, there are individuals with ASD who are motivated by the fact that mathematical operations have clear rules and if they follow them, they always get the same results. Maths becomes a popular topic; students are motivated by the fact that they can count and this can even turn into a typical interest.

Research Methodology

The aim of the research was to compare the mathematical problem-solving processes of students with ASD with those of their neurotypical classmates and to show, through qualitative analysis, whether an "autistic cognitive style" can be distinguished and whether students with ASD can be expected to have good mathematical abilities because mathematics – being an exact science – suits their way of thinking better (see e.g. Baron-Cohen, 2009; Fitzgerald & James, 2007; Frith, 2003). In order to meet the objectives of the research investigation, the following research questions were established:

- **Do students with ASD use qualitatively different problem-solving procedures than their peers when working on mathematical problems?**
- **Is it possible to find similar particularities in the problem-solving procedures that are specific to the students with ASD included in our research and which are different from their neurotypical peers?**
- **Are the students with ASD in our sample more successful at problem solving than their peers?**
- **Do we see mathematical talent in the students we studied?**
- **Can we find qualitative differences in the processing of most of the problems assigned between the boys and girls in our sample?**

As is evident, we focused on whether the students with ASD in our research sample found a solution to the mathematical problem at hand that was original and different from their peers and from possible commonly proposed approaches. It was also important to determine whether the students with ASD would perform in the same or similar ways, i.e. whether we could detect a similar specific style of problem-solving processes in several students with ASD.

For the purposes of qualitative research, we chose the following research method: solving mathematical problems, reflection on the procedure, interviews with the students about the problem-solving process, and observation of the students and analysis of the notes as well as graphical processing of the problems. For the sake of completeness, we further enhanced our observations by interviewing mathematics teachers. Examples of the mathematical problems are presented in Appendices 1 and 2.

Research Sample

The research itself took place at an elementary school and a grammar school in Prague, where they have been integrating students with ASD for a long time now, which means the teaching staff have extensive experience with teaching and supporting students. In the preparatory phase, we focused on the selection of students for the research investigation. Since we chose a qualitative approach, we paid a great deal of attention to this in order – as pointed out by Hendl (2006), for example – to meet the requirement of the suitability and appropriacy of informants for the research inquiry. We selected students in grades 6 and 9 for the sake of data variability and diversity. It was important for us that the students were at a developmental level where they would be able to reflect and report on their practices. In each grade group, we first administered a mathematics test (the Mathematical Kangaroo) as one of the criteria for "matching" pupils in mathematics lessons.

Subsequently, all the students with ASD in the class were approached and matched with classmates on the basis of age (the age difference could not be more than six months) and mathematics test scores. After an informed consent had been obtained from the parents and students, each student with ASD was matched with a boy and girl from the same class whose age and performance met the criteria (see Table 1) and who also did not have any specific learning difficulties (ADHD, learning disabilities, developmental dysphasia, etc.). Although we were aware of the specific diagnoses of the students with ASD, we did not consider these further in our data analysis. We assumed that, according to experts (cf. Frith, 2003; Šporclová, 2018), all individuals with ASD exhibit core difficulties, differing only in the degree and severity of their manifestations.

The Ethical Context of the Research

As mentioned earlier, an informed consent was obtained from the informants'

Table 1. Participants

Name	Grade	Age	Sex	Mathematical Kangaroo	Block Design WISC III	ASD
ADAM	**6**	August 2003	M	79	62	YES – childhood autism
AGÁTA	**6**	November 2003	F	57	44	NO
ALEŠ	**6**	August 2003	M	76	54	NO
BRUNO	**6**	March 2004	M	89	65	YES – Asperger syndrome
BEDŘICH	**6**	December 2003	M	88	61	NO
BARBORA	**6**	March 2004	F	87	66	NO
CYRIL	**6**	June 2003	M	95	56	YES – Asperger syndrome
ČENĚK	**6**	August 2003	M	103	58	NO
CECÍLIE	**6**	September 2003	F	91	56	NO
DAVID	**9**	September 2000	M	27	63	YES – childhood autism
DOMINIK	**9**	November 2000	M	32	59	NO
DITA	**9**	July 2000	F	29	39	NO
ELIÁŠ	**9**	June 2001	M	51	64	YES – Asperger syndrome
EDA	**9**	March 2001	M	54	61	NO
EVA	**9**	January 2001	F	56	69	NO
FILIP	**9**	June 2000	M	41	51	YES – Asperger syndrome
FERDINAND	**9**	October 2000	M	47	60	NO
FRANTIŠKA	**9**	October 2000	F	52	60	NO

legal guardians before their inclusion in the research sample, via which consent was also obtained for the audio recordings of all sessions. The informants themselves were then briefed about the research process at the beginning of the individual sessions and their verbal consent was obtained. They were also informed that they could end the session at any time – while working on a single problem, they could cancel a whole session or, alternatively, they could withdraw from the research altogether. However, no one took advantage of this option.

Individual Sessions

During the individual sessions, the students were presented with mathematical problems to solve and were asked to describe how they arrived at their results. The mathematical problems were selected from freely available mathematical Olympiads, Mathematical Kangaroos, and the entrance exams to grammar schools (see Appendix for an overview). It was not essential for the aim of the investigation that students got the solution right; however, we were aware of the fact that if the problems were too challenging, it might affect the students' motivation to cooperate further. The session had no time limit.

Once a problem was completed, we did not reflect on whether the procedure and the results were correct. We asked about the procedure and the results they had reached. We did not analyse the errors or other possibilities of solving the problem. The ninth-graders solved six problems and the sixth-graders solved five problems (see Sotáková, 2019).

During the session, we kept notes of our observations, which we called testing notes. We recorded the students' behaviour during the session, and for the students with ASD, we also focused on the manifestations associated with autism spectrum disorder, whether these were manifestations in speech, social communication and behaviour, or unusual behaviour.

Data Processing

As is apparent from the previous section, quite a large amount of data was collected during the course of the research. The descriptions of the students' solutions were transcribed, and each student's progress was recorded, using transcription of the recordings into an abbreviated version, where we added our own notes and an analysis of the notes or graphical solutions to the problems. In this first analysis, we tried to distinguish areas, i.e. categories within which we could further compare the procedures of the students with ASD with their neurotypical classmates. Our aim was to reduce the data obtained to a form where we would be able to work with it further and identify individual aspects of the cognitive processing of each student.

The categories that we then divided and used to analyse each student's practices were as follows: **description of the solution to the problem or parts of the problem; comparison with the recommended solving procedure; analysis of the written solution to the problem/sketch; common and divergent points of the procedure and solution; manifestations of core problems associated with ASD; analysis of errors/**

critical points for successful problem solving; solution time.

Results and Discussion

The results of the study showed that finding a specific cognitive style in students with ASD that differentiates them from their classmates is difficult. They support the above findings by emphasizing that even if it can be said that certain manifestations of core difficulties can be traced in many individuals with ASD, the variability among them is still great and we would have to resort to simplifications that, given the distinctiveness of the individuals, would be uninformative or even misleading.

Summary of Sixth-grade Results

The results were mixed for the sixth-grade students. Although we detected at least partly the same problem-solving procedures for all the students with ASD as in their paired classmates in most problems (often corresponding to the recommended problem-solving procedure), in other respects, however, the processes of the students with ASD were different, both in terms of success in solving the problems and in terms of the procedures used. The least successful of the three students with ASD was Adam, who was not able to solve any of the problems. He was an elementary school student who also scored the lowest of the students with ASD in the mathematics placement test. It can therefore be assumed that his performance is consistent with his ability, which is at a lower level than Bruno and Cyril. It is, however, worth noting that Adam achieved the highest score of all three students in the Block Design subtest (WISC III). He got stuck on a specific part of a problem or disregarded the whole problem in most of the assignments. For this reason, we discussed the possibility of a significant weakening of his central coherence since his practices of ***focusing on a certain aspect of the problem***, on a certain detail, especially when the solution involved several steps or levels, corresponded to the performance profile as described, for example, by Happé (1999) or Frith (2003).

We observed a match in the solution in only one case, namely in the second problem as solved by Adam and Bruno, when both of them arrived at the same result. However, even here, their steps were not exactly the same; while Adam did not work with the sketch at all and directly converted the numbers from the assignment into mathematical operations, Bruno first tried to make a sketch (he made two) and only when he did not know how to proceed further did he come up with a simplifying solution.

Bruno was able to solve the first problem successfully, and for the second, third, and fourth problems he was able to follow the recommended solution at the beginning but eventually reached a critical point, which he could not handle. His justification of the solution was interesting and here again, we found similar tendencies to Adam's approach. Neither of them doubted their solutions, they did not return to the wording of the assignment, they did not try to structure the problem by writing it down, and they argued for the result they reached. (Adam further supported this by making comments about his good mathematical skills.) Bruno showed a different approach only in the fifth problem. It was noticeably too difficult for him and he could not form an idea of the problem to be solved. This led to him returning to the original assignment for the first time, expressing his doubt about the solution (*"It's no use, I'll get it wrong anyway."*) and ending the session. For Bruno and Adam, an obvious tendency towards quick, direct solutions was also apparent. This is also evident in the solution times, which were shorter than those of their peers – in some problems significantly so.

Cyril was the most successful in solving the problems of the students with ASD who were observed. He solved three problems correctly, failing to solve only the second and fourth problems. Even in these problems, however, he started out correctly but in the second problem he was not able to take into account all the parameters of the assignment (we cannot say that he focused on only a certain part of the problem), which led him to an incomplete solution in which he made a numerical error on top of that. He thought that he had followed the correct procedure. For the fourth problem, he showed rigidity in his thinking when he was unable to "traverse" a critical point and consider a different solution procedure and repeatedly returned to the same point. He was the only sixth-grade student with ASD to revise his procedures, returning to the assignment's wording, although here, he was simply unable to grasp the problem from a different angle. His different approach during the research investigation was also reflected in the problem-solving times. Unlike the other sixth-grade students with ASD, we observed longer solution times in Cyril than in his peers. His problem-solving procedures were comparable both to the recommended procedures and to those of his neurotypical classmates, especially Čeněk. On the other hand, we cannot ignore the fact that he showed some of the behavioural manifestations of core ASD difficulties (fascination with the voice recorder, slow work pace, motor restlessness) and that visual support in the problem solving was important for him (he used pictures and sketches spontaneously for most problems). He had the most difficulty with the fourth problem, which was an equation without the possibility of graphical representation. Cyril finished at a point that was critical for most of the sixth-grade students. This was not surprising as the problem was difficult for the students. However, we were intrigued that even after being given a hint and after crossing out the original solution, Cyril proceeded in exactly the same way over and over again. We attribute this to a certain rigidity in thinking – an inability to abandon a given procedure and try to look at the solution from a different angle. **Limited flexibility in thinking** is thus certainly one of the factors affecting performance in mathematics (see Donaldson & Zager, 2010).

In the sixth grade, the students with ASD were comparable to their paired peers in the course of problem solving, although we found some qualitatively different procedures. However, this was not the case for the majority of the assignments and, in addition, even the neurotypical students were qualitatively different in their procedures, particularly when they found the problem difficult and did not know exactly how to proceed. We detected the most irregularities in Adam, in whom there was a significant difference in his performance in the mathematical problems where he was unable to solve even one correctly while achieving an above-average result in the Block Design subtest (see Table 1).

Summary of Ninth-grade Results

The results of the students with ASD in the ninth grade were also variable. Each of the ninth-grade students was quite distinctive and it was difficult to find similarities between them, although they were qualitatively more different from their classmates in their procedures than the sixth-grade students. In this section, we will therefore attempt to compare their practices and detect areas of expression typical of students with ASD.

In terms of problem solving, the most successful student with ASD was Eliáš. Although his procedures were unusual and could be characterized by a tendency to convert the assignment immediately into a mathematical operation, he was able to proceed appropriately using adequate mathematical formulae when he understood the assignment. For problems that were difficult for the imagination and in terms of conceptualization, he typically converted the problem into an equation, which he then tried to simplify. For him, the equation represented the notation of the problem, giving him the structure needed for understanding, although from our perspective it was rather confusing, as he ended up with equations with two or three unknowns, which he could not replace with anything. Filip and David also had problems in forming an idea of the nature of the problem (this was also present in the neurotypical students) but they were qualitatively different. While Filip was able to form an idea for some of the problems, even though he was not able to solve them (for example, the first problem, where he knew how to proceed, but because of a repeated numerical error he did not arrive at the result), for David, we noted that the problems were beyond his capabilities, i.e. he was not able to conceptualize the problem or understand its essence.

Problems with task conceptualization are certainly related to the students' mathematical abilities but may also be influenced by **impaired central coherence or deficits in executive functions** as described by Thorová (2006). Parameters supporting weak central coherence (Happé, 1999) were detected only in David; the other boys showed results comparable to their classmates.

We noted commonalities particularly in the boys' comments and descriptions of their own practices. David and Filip referred to formulae they did not know or had forgotten when they did not know the procedure. David: *"But it doesn't make any sense to me now, word problems like that, I'd have to have it in my head what formulae to put in, how to count them under each other, to reduce them, divide them, just..."* And Filip: *"I'm sorry, but the last time we learned volume conversions was in the first or second grade and I kind of don't remember it."* Comments unrelated to problem solving that described the boys' physiological or psychological state were also frequent. In our view, this was to relieve the boys' frustration when they did not know how to proceed or realised that the problem was beyond their capabilities. David's comments were mainly limited to *"I don't know... I don't get it."* Eliáš remarked, for example, *"You're checking if it's still being recorded; well, that's great..."* or *"I don't understand at all how teachers can give such long speeches because when I talk for a long time my mouth gets so dry, and especially when they have to shout at our noisy class..."* Filip expressed his frustration in comments that related to the authors or characters in the problem: *"Honestly, I think it's bullshit because they can't even physically fit in there, I mean, like, maybe into the new ones, yeah OK, but..."* Here, he contradicted the factual logic of the assignment but he also turned his attention to the persons in the assignment: *"Mr. Celer, holy shit, Comrade Celer."* We did not encounter such utterances among the neurotypical students. Disregard for social rules and difficulties in self-regulation of behaviour are widely described in the literature on individuals with ASD (Thorová, 2006; Wing, 2002; Attwood, 2005). **Deficits in executive functions** and misunderstanding of social rules are cited as the reasons. The described manifestations were exacerbated in moments of emotional strain (stress, problem difficulty, misunderstanding); similar manifestations appeared in the sixth-grade students with ASD but not in the neurotypical students.

On the basis of these results, we can answer the research questions posed above in the paragraphs that follow. We asked whether it is possible to confirm that children with ASD **use qualitatively different problem-solving procedures than their peers when processing mathematical problems**. Our results only partially confirmed such a conclusion. In the sixth grade, the students with ASD mostly proceeded similarly to their peers in at least part of the assignment. Bae, Chiang, and Hickson (2015) reached similar conclusions, that is, that word problem solving for students with ASD is influenced to a large extent by their reading comprehension skills. The observed specifics can be related to the theory of impaired central coherence. In the ninth grade, the students' solutions were highly individual, both for the students with ASD as well as their classmates. This was particularly evident in problems that were difficult for the students to conceptualize, i.e. to form an idea of the situation in the problem.

During the course of the research investigation, we did not observe **any similarities in the problem-solving procedures among the observed students with ASD that were specific and different from their neurotypical peers**. We did, however, trace certain tendencies in the approach to problem solving. The first of these was a tendency towards rapid operationalization, i.e. translating the problem into mathematical operations. Most of the students with ASD (unlike their classmates) did not elaborate the notation but worked directly with calculations or rewrote the assignment into mathematical expressions (such as Eliáš using equations). The quick solution then sometimes led to a focus on a particular line of the problem regardless of the assignment (Adam, Bruno). This practice is usually attributed to weakened central coherence. However, on the evidence of our results, we cannot support Baron-Cohen's conclusions about an „autistic" cognitive style (2005). The tendencies described were not strong enough to claim that we found cognitive practices specific to students with ASD.

We also looked at whether the students with ASD in our sample would be significantly **more successful than their peers** in their solutions. This was not confirmed; we can talk about similar performance or even the opposite trend. In the sixth grade, the students' success varied but we can say that it was comparable between the students with ASD and their neurotypical classmates. In the ninth grade, performance was comparable between the students with ASD and at least one of their neurotypical classmates'. David approached problem solving similarly to Dita but Dominik was more successful. Filip, on the other hand, performed comparably to Ferdinand, while Františka performed better. We thus did not observe mathematical talent in our study.

Although we did not primarily focus on the social behaviour and communication of the students with ASD (we considered them to be core areas, i.e. we assumed that they would become apparent during testing), we approached them as an important source of information in the data analysis. For the children with ASD, we were able to observe not only their emotional dispositions but also the contents of their minds through comments given during the problem solving or their stereotypical behaviours. In contrast to their neurotypical classmates, they often described their physiological dispositions, thoughts, and assessments of the problems and their authors. The students with ASD fulfilled the criteria for **executive function deficits** (Ozonoff, Pennington, & Rogers, 1991), not only in the behavioural manifestations described but also in their approach to problems. Some had difficulty returning to the original assignment, revising their procedures, or planning and structuring their procedures in a way that led to successful problem solving (Adam, Bruno, Eliáš, Filip).

Another interesting aspect was the developmental shift we observed among the students with ASD in each grade. While in the sixth-grade students, we repeatedly saw that their procedure was detached from the assignment – that they were solving something else – they did not doubt their result (the exception was Cyril, who was able to revise his procedure and work with the assignment), the ninth-grade students, on the contrary, often stated that their solution was probably not correct and returned to the original assignment. Although we cannot generalize from such a small number of students, in whom the aspect in question may be due to the personal characteristics of the students, the above tendency supports the opinions of experts that emphasize the developmental aspect in individuals with ASD. They state that it is not possible to take a simplistic view of their specificities and deficits, as they may change significantly during development in interaction with the environment (Thorová, 2006; Šporclová, 2018).

The results of our study are therefore inconclusive. We can summarize that the performance of the students with ASD is similarly variable to that of their neurotypical classmates. Although we observed some specificities in behaviour and communication, we can speak of tendencies in terms of cognitive processing. The observed tendencies were consistent with the **theory of impaired central coherence and the theory of deficits in executive functions** but our results did not support specific problems in the theory of mind.

Conclusion

The results of our study show that it is not possible to conclude that students with ASD have a specific style for solving mathematical problems. However, we did reveal some tendencies in cognitive processing. The first of these was a ten-

Appendix 1. Example of mathematical problems – sixth grade

Problem	Procedure
Ruda got an alarm clock for his birthday. He was happy with it and adjusted it according to the exact time. Since then, every morning, when he got up (Saturdays, Sundays, and the holidays are excluded), he pressed the button that illuminated the dial for exactly four seconds. At the same time, he noticed that the alarm time was stopped while the button was pressed. Otherwise, the alarm does not delay or accelerate at all. On the afternoon of December 11, Ruda looked at his alarm clock and found that he was showing exactly three minutes less than he should. When did Ruda get this alarm clock?	1. First we find out how many times Ruda pressed the button, we divide the total alarm delay by the number of delays per day. So Ruda pressed the button a total of 45 times (180: 4 = 45). 2. We count down 45 days from December 11: In December (from 11th to 1st) it is 11 days, November (from 30th to 1st) is 30 days, which is a total of 41 days. We need to count down another 4 days in October: 31st, 30th, 29th, 28th. Ruda pressed the lighting button for the first time on October 28. He received the alarm the day before, i.e. October 27.

dency towards **quick, direct solutions.** The students with ASD were more likely than the typically-developing students to attempt immediate operationalization of the problem. For the sixth-grade students, this often resulted in them not grasping the assignment comprehensively but focusing on a particular aspect of the problem. They then presented their solution as the result, and although it was usually incorrect, they did not doubt it. They were unable to **revise their procedures or go back to the original assignment.** For the ninth-grade students, the outcomes were different. Although some of them also showed a tendency to work quickly, they were able to reflect on their procedures and solutions, going back to the assignment, realizing that the result at which they arrived was incorrect. We could therefore observe a certain development in their self-evaluation and the evaluation of their procedures, which was also reflected in the problem-solving process.

The results did not support specific differences in the processes of the students with ASD and their typically-developing classmates. All the students' performances were variable; sometimes their procedures intersected more and sometimes less. Nor did we find significant differences between the boys and girls in our sample. Differences in problem-solving procedures were more likely to be due to the mathematical abilities of individual students; however, we did observe some specificities in students with ASD that can be linked to theories of impaired central coherence and impaired executive functions. In conclusion, it should be stated that the mathematical abilities of students with ASD cannot be considered to be generally above aver-

Appendix 2. Example of mathematical problems – ninth grade

Problem	Procedure
A 30-member group came to a hostel. It fully occupied the same number of quadruple and double rooms. A person in a double room spent the night for 300 CZK per night, a person in a quadruple room for 200 CZK per night. Calculate how many rooms the group occupied. Calculate the price of accommodation for the whole group in one day.	1. We can solve this as an equation of one unknown: 2x + 4x = 30, the result is 5, together ten occupied rooms (five double, five quadruple) 2. We know that the group will occupy 5 + 5 rooms; in five double rooms there are ten people, i.e. 10x300 and in five quadruple rooms there are twenty people, i.e. 20x200, then you just need to add 3000 + 4000. The price for a group per day is 7000.

age; it is always necessary to consider the specific characteristics of the student and the severity of the manifestations of autism spectrum disorder.

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PhDr. Hana Sotáková, Ph.D.
Faculty of Education, Department of Psychology
Charles University
hana.sotakova@pedf.cuni.cz